# Problem Set 3: Signaling

PLEASE SUBMIT BEFORE CLASS ON NOVEMBER 9

**Question 1**

Consider a game in which, first, nature draws a worker's type from some continuous distribution on $[\underline{\theta}, \bar{\theta}]$. Once the worker observes her type, she can choose whether to submit to a costless test that reveals her ability perfectly. Finally, after observing whether the worker has taken the test and its outcome if she has, two firm bid for the worker's services.

Show that in any subgame perfect Nash Equilibrium of this model all worker types submit to the test, and firms offer a wage no greater than $\underline{\theta}$ to any worker not doing so.

**Question 2**

Consider a market for loans to finance investment projects. All investment projects require an outlay of \$1. There are two types of projects: good and bad. A good project has a probability of $p_G$ of yielding profits of $\Pi > 0$ and a probability $1 - p_G$ of yielding profits of zero. For a bad project, the relative probabilities are $p_B$ and $1 - p_B$, respectively, where $p_G > p_B$. The fraction of projects that are good is $\lambda \in (0, 1)$.

Entrepreneurs go to banks to borrow the cash to make the initial outlay (assume for now that they borrow the entire amount). A loan contract specifies an amount $R$ that is supposed to be repaid to the bank. Entrepreneurs know the type of project they have, but the banks do not. In the event that a project yields profits of zero, the entrepreneur defaults on her loan contract, and the bank receives nothing. Banks are competitive and risk neutral. The risk-free rate of interest (the rate the banks pay to borrow funds) is $r$. Assume that

$$p_G\Pi - (1 + r) > 0 > p_B\Pi - (1 + r)$$

where $1 + r$ is the cost to the bank of borrowing \$1. This cost the bank pays for sure if it gives a loan to the entrepreneur, however, only with probability $p_i$ the bank gets back his loan, thus, expected profits of the bank that sponsors project with $p_i$ is

$$p_iR + (1 - p_i)0 - (1 + r)$$

and the profits of the entrepreneur with the project $i$ is

$$p_i(\Pi - R) + (1 - p_i)0$$

1. Find the equilibrium level of $R$ and the set of projects financed. How does this depend on $p_G$, $p_B$, $\lambda$, $\Pi$ and $r$?

2. Now suppose that the entrepreneur can offer to contribute some fraction $x$ of the \$1 initial outlay from her own funds, $x \in [0, 1]$. The entrepreneur is liquidity constrained, however, so that the effective cost of doing so is $(1 + \rho)x$ where $\rho > r$.

   (a) What is an entrepreneur's payoff as a function of her project type, her loan-repayment amount $R$ and her contribution $x$?

   (b) Consider the following game. First, the entrepreneur makes an offer that specifies the level of $x$ she is willing to put into a project. Second, banks respond by making offers specifying the level of $R$ they would require. Finally, the entrepreneur accepts a bank's offer or decides not to go ahead with the project. Describe the separating perfect Bayesian equilibrium of this game. How does the amount contributed by entrepreneurs with good projects change with small changes in $p_B$, $p_G$, $\lambda$, $\Pi$ and $r$?

   (c) How do the two types of entrepreneurs do in the separating equilibrium of part 2(b) compared with the equilibrium of part 1 of this question?