PLEASE SUBMIT BEFORE OCTOBER 19 CLASS

EC135 - Problem Set 2

Question 1

One way to construct preferences over lotteries with monetary prizes is by evaluating each lottery p on the basis of two numbers:

- the expectation of lottery p, $E(p) = \sum z \cdot p(z)$
- variance of lottery p, $var(p) = E((z - E(p))^2) = \sum (z - E(p))^2 \cdot p(z)$

Such a construction may or may not be consistent with vNM assumptions.

1. Show that the function $u(p) = E(p) - \frac{1}{4} var(p)$ induces a preference relation that is *not* consistent with the vNM assumptions. (For example, consider the mixtures of each of the lotteries $[1]$ and $\frac{1}{2}[0] \oplus \frac{1}{2}[4]$ with the lottery $\frac{1}{2}[0] \oplus \frac{1}{2}[2]$.)
2. Show that the utility function $u(p) = E(p) - (E(p))^2 - var(p)$ is consistent with vNM assumptions.

Question 2

Consider two risk averse individuals with initial wealth W.

Let u and v be the vNM utility functions of two individuals (identical except to attitude towards risk), where u is *more risk averse* than v.

Assume that u and v are twice continuously differentiable and u', $v' > 0$.

Each individual faces a random damage: with probability $q \in (0, 1)$, there will be damage in the amount X, where $0 < X < W$.

An insurance company is willing to sell insurance for this risk: a coverage I satisfying $0 \leq I \leq X$, will cost

$$P(I) = (1 + m)qI \qquad \text{where } m > 0$$

Each of the individuals buys a positive optimal insurance coverage $I_u^\star$ and $I_v^\star$.

Show that the more risk averse individual buys more insurance, i.e., that the optimal insurance coverage satisfies

$$I_u^\star > I_v^\star$$