PLEASE SUBMIT BEFORE OCTOBER 10 CLASS

EC135 - Problem Set 1

Question 1

Show that continuity axiom (C) implies Archimedean axiom (C*).

Question 2

Consider the following preference relations on lotteries. For each preference relation check carefully whether it satisfies independence axiom (I) and continuity axiom (C).

1. *Expected Utility:* A number $v(z)$ is attached to each prize, and a lottery p is evaluated according to its expected v, that is, according to $\sum_z p(z)v(z)$. Thus,

$$p \succeq q \text{ if } U(p) = \sum_{z \in Z} p(z)v(z) \geq U(q) = \sum_{z \in Z} q(z)v(z)$$

2. *Comparing most likely prize:* The decision-maker considers the prize in each lottery that is most likely (breaking ties in some arbitrary way) and compares two lotteries according to a basic preference relation over Z.

3. *The size of the support:* The decision-maker evaluates each lottery by the number of prizes that can be realized with positive probability, that is,

$$p \succeq q \text{ if } |supp(p)| \leq |supp(q)| \text{ where } supp(p) = \{z | p(z) > 0\}$$

Question 3

Consider the following four lotteries:

$$L_1 = 0.25[\$3000] \oplus 0.75[\$0] \qquad L_2 = 0.2[\$4000] \oplus 0.8[\$0]$$

$$L_3 = [\$3000] \qquad L_4 = 0.8[\$4000] \oplus 0.2[\$0]$$

Show that the decision-maker who has the following preferences, $L_2 \succ L_1$ and $L_3 \succ L_4$, violates Independence axiom.

Question 4

Consider a decision-maker (DM) with wealth W who is presented with a 50-50 gamble that can either yield a gain of \$200 or a loss of \$100. Suppose that for any W, the DM rejects this gamble, but yet there exists some wealth W^* at which he is willing to accept two such gambles executed back-to-back. Show that this DM cannot be an expected utility maximizer who strictly prefers more money to less.